2.1)

Behauptung : $t_j = u_j a + v_j b$

Induktionsanfang : $t_0 = 1 \cdot a + 0 \cdot b = a$

$t_1 = 0 \cdot a + 1 \cdot b = b$

Induktionsschritt : $t_{j+1} = t_{j-1} - q_{j+1} t_j$ mit $q_{j+1} = t_{j-1}\ div\ t_j$

$= u_{j-1} a + v_{j-1} b - q_{j+1}(u_j a + v_j b)$

$= (u_{j-1} - q_{j+1} u_j) a + (v_{j-1} - q_{j+1} v_j) b$

$= u_{j+1} a + v_{j+1} b$ q.e.d.

2.2)

$arla \mathrel{\hat{=}} [0]_{29}, [17]_{29}, [11]_{29}, [0]_{29}$

$dplw \mathrel{\hat{=}} [3]_{29}, [15]_{29}, [11]_{29}, [22]_{29}$

$$\begin{pmatrix} [3]_{29} \\ [15]_{29} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} [0]_{29} \\ [17]_{29} \end{pmatrix}$$

$$\begin{pmatrix} [11]_{29} \\ [22]_{29} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} [11]_{29} \\ [0]_{29} \end{pmatrix}$$

$\Rightarrow$

1.) $[3]_{29} \equiv [17]_{29} b$

2.) $[15]_{29} \equiv [17]_{29} d$

3.) $[11]_{29} \equiv [11]_{29} a \quad \Rightarrow a = [1]_{29}$

4.) $[22]_{29} \equiv [11]_{29} c \quad \Rightarrow c = [2]_{29}$

zu 1.) $[3]_{29} x \equiv [17]_{29} bx \equiv 1 \qquad ggT(3,29) = 1 \qquad [3]_{29}^{-1} = [10]_{29}$

$[3]_{29} \cdot [10]_{29} \equiv 1 \equiv [17]_{29} \cdot [10]_{29} b$

$b = [7]_{29}$

$d = [6]_{29}$

$$\Rightarrow \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} [1]_{29} & [7]_{29} \\ [2]_{29} & [6]_{29} \end{pmatrix}$$

$$\det \begin{pmatrix} [1]_{29} & [7]_{29} \\ [2]_{29} & [6]_{29} \end{pmatrix} = [6 - 14]_{29} = [21]_{29} \qquad \Rightarrow [21]_{29} x \equiv 1 \ \Rightarrow x = [18]_{29}$$

$$\begin{pmatrix} [1]_{29} & [7]_{29} \\ [2]_{29} & [6]_{29} \end{pmatrix}^{-1} = [18]_{29} \begin{pmatrix} [6]_{29} & [-7]_{29} \\ [-2]_{29} & [1]_{29} \end{pmatrix} = \begin{pmatrix} [21]_{29} & [19]_{29} \\ [22]_{29} & [18]_{29} \end{pmatrix}$$

$\Rightarrow$ “strike_at_noon!karla“